

Circuit Theory

— and —

Design

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Preface

The majority of devices of concern to the electrical and electronics engineer can, at least to a first order, be described as linear. Virtually all systems have characteristics that vary in some manner with frequency. Unquestionably, the most powerful technique for understanding and controlling linear, frequency-dependent systems is that afforded by the poles and zeros of network functions. In addition to the general conceptual and analytical tool which they provide, their manipulation also yields an exceptionally effective tool for network design. In electric and mechanical circuits both with and without energy sources, the pole-zero approach tends to make analysis and design one and the same; so powerful is the technique in providing understanding (analysis) that only a slight change in viewpoint leads to design (synthesis).

Although pole-zero methods have been employed by high-level systems designers and network synthesis people for several years, the technique has not become nearly as widespread as is justified. In fact, its greatest value appears to lie outside the realm of formal network synthesis, where it has apparently resided all this time. Because a descriptive and relatively thorough text has not been available, the student, as well as his teacher, has remained partially unaware of pole-zero methods. This book has been written to help fill the void. It includes considerable information and design procedures of practical significance and discusses a wide variety of specific devices. As background, the student should have had calculus and if possible a course in elementary differential equations, although the latter is not necessary. In addition, the student should have taken the usual sophomore and junior courses on circuit theory and preferably an introductory course on vacuum tubes. To facilitate the use of this book as a text, a number of problems have been provided for each chapter. To a large extent, they relate to practical system design. In addition, the problems present many useful formulas and concepts.

It has been my express intent to write this book so that it might prove effective in undergraduate courses without compromising its value in

graduate study. Although the introduction of poles and zeros in other than a cursory fashion into the undergraduate curriculum is somewhat revolutionary at this time, I nevertheless believe it is inevitable. The problem involved in carrying out my intent was therefore reduced to the manner of organizing the material. In order to point out how I feel the book can best be used, a brief résumé of its content is warranted.

Chapter 1 is a relatively thorough and generalized review of steady-state circuit analysis. The student may have been exposed to much of this material, although some of it, such as determinant manipulation, may be new. I have adopted the less common convention of associating the negative sign with all mutual terms in node and loop equations, which has many advantages. The student learns the form of a general system of equations from relatively simple examples, which always show the negative sign. It is cumbersome and confusing to the student to define a fiction such as coimpedance in order to write plus in front of off-diagonal terms. In order to apply determinantal methods, the negative sign must ultimately be set down anyway.

Poles and zeros are first introduced in Chapter 2, employing a strict phasor approach, and the behavior of functions is emphasized in terms of their poles and zeros. In Chapter 3, the relationships between networks and poles and zeros are discussed. In addition, a fairly extensive introduction to transient calculations is given, with a Heaviside approach. The concepts developed in Chapters 2 and 3 are freely used in the balance of the book.

Chapter 4 deals with R - L , R - C , and L - C networks and their canonical forms, and also dwells at considerable length on the extremely important concepts of impedance and frequency normalization and frequency transformations. By assuming a knowledge of normalization and transformation throughout the balance of the book, it has been possible to make the later parts more compact and less cluttered than is normally possible.

Chapter 5 is a fairly detailed study of maximally flat, linear-phase, and other functions, maintained fairly independent of specific networks. This chapter is, in essence, a treatise on the approximation problem. Some of the functions developed in Chapter 5 are often presented at later points in the book without additional discussion.

Chapter 6 introduces some of the more important topics from modern network synthesis, for example, a simplified form of the Darlington procedure. It is not intended as a substitute for a formal graduate course in modern network theory, although it does provide a good starting point.

Chapters 7 and 8 are devoted to the classical theory of image matching in somewhat more mathematical detail than is customary in undergraduate texts. A reliance on a knowledge of frequency transformations and normalization has greatly reduced the amount of space necessary to cover this material adequately.

Chapter 9 is a study of vacuum-tube linear equivalent circuits and some special phenomena associated with tubes. Although the level of this chapter is somewhat lower than the rest of the book, its inclusion is felt to be justified for reference, completeness, and to facilitate the use of the text in the undergraduate curriculum. Chapters 7 and 8 serve a similar purpose in some respects.

Chapters 10 and 11 go into the design of low-pass and band-pass amplifiers in some detail, with the aim of developing relatively simple methods for designing amplifier systems which are today considered rather sophisticated. Poles and zeros are employed extensively in these chapters. In addition, the step-function response of low-pass amplifiers is studied in more than a cursory fashion.

The final three chapters are devoted to feedback devices. Chapter 12 is concerned with feedback amplifiers, and emphasizes the precision design of the closed-loop transfer function. Chapter 13 takes up linear oscillators, generalized to a considerable extent in relation to the associated circuitry. Chapter 14 is concerned with the functions of interest in servomechanism theory and design. As in Chapter 12, precision design is emphasized. This chapter, along with the rest of the book, provides a much more thorough background for the serious study of linear servomechanisms than is usual.

As stated, I have tried to organize the subject matter so that the book can be used at either the graduate or the undergraduate level. How I feel the book can best be used at the graduate level will first be discussed.

The usual graduate program has three distinct courses related to circuits: a one-year study of formal network theory, a one-year consideration of electronic circuit design, and a one-semester course on transient analysis. In this book I have tried to combine practical linear circuit design with much of the material contained in formal network theory. No attempt has been made to integrate a detailed study of transients into the presentation, although the material in Chapter 3 is adequate for most practical problems. Rather, it is felt best to leave this to an independent course. In addition, most of the formality and rigor of network theory has been supplanted with more readily understandable pole-zero and phasor concepts.

This book can be covered in a one-year course, especially if Chap-

ters 1, 4, 7, 8, or 9 are treated as review material. In addition to this book, the first-year graduate program can be made complete along the lines of circuit and network theory by adding the usual formal course on Fourier and Laplace theory, a one-semester course on nonlinear and other special aspects of electronic circuits, and, finally, a one-semester treatment of formal network theory, which is better appreciated by a student who has the first half of this book as background than by one with an earlier formal network theory course. A student of network and circuit theory has always been hampered by the lack of an easily grasped pictorial representation; poles and zeros interpreted as phasors can furnish this lack.

The possible adoption of this book in the undergraduate program has been my fondest hope. I am convinced that it can be used effectively, providing suitable instructors are available to teach it. Of course, it is always hard to modify an undergraduate curriculum, which is a difficulty that must be accepted. However, the book has been written with this difficulty in mind. The first eight chapters, with the probable exception of Chapter 6, can be introduced most easily through a modification of the customary course on classical filter theory, which normally has about a year of circuit analysis as a prerequisite. Chapters 1, 4, 7, and 8 include all the material usually taught in such courses, which means that only Chapters 2, 3, and 5 represent new material. If the first half of the book serves as a text, the introduction of the second half into the curriculum is no great problem; Chapters 9 through 14 can be covered in one semester.

Unfortunately, the new graduate does not appear to know much circuit design, and may even have trouble with relatively simple analysis problems. One reason for this is the great bulk of concepts forced upon him without a detailed study of any one, which is a recent malady arising from the tremendous expansion of frontiers. Teaching too many concepts is questionable philosophy; it is much easier to learn concepts after graduation than to learn techniques of analysis and design involving mathematical and even simple algebraic manipulations. This is of particular concern in the field of linear circuit theory and design, which constitutes what is probably the engineer's most important special knowledge. His lack of training in circuit theory is compounded because the classical approach does not leave him with a thorough and deep-seated understanding of networks. The pictorial approach given by poles and zeros is particularly suited to providing the needed understanding. However, practice and repeated application to design problems are necessary if the student is to retain his knowledge; in other words, a cursory introduction to the subject is not adequate.

The undergraduate and many graduates find it difficult to grasp circuit theory thoroughly if all proofs and manipulations are purely mathematical; they need some picture in their minds. For this reason, the formal justification of poles and zeros with Laplace, Fourier, or comparable mathematical disciplines has been ignored. In addition, analogs such as electrolytic tanks and rubber membranes have been ignored; I do not feel they give any more understanding than that obtainable from a simple phasor approach. Wherever possible, the phase and amplitude characteristics of a circuit have been implied with a simple pole-zero diagram in hopes that the student will begin to think and visualize in these terms.

An effort has been made to show that the variable p has many interpretations, all of which can be considered simultaneously; frequency variable, complex variable, derivative operator, and Heaviside (or Laplace) operator. The result of this emphasis on the multitudinous nature of p has been a loose functional notation for voltages and currents. It would be unfortunate to restrict the interpretation of a voltage by writing a symbol such as $E(j\omega)$, which is normally reserved for the steady state.

I will no doubt be accused of using symbols in a few places that do not conform to certain standards set down a number of years ago. The first is the way I define plate and grid voltages. I have not flaunted standard notation; rather, I have avoided it. In all but the simplest vacuum-tube equivalent circuits, all node voltages must be taken with respect to the reference node if utter confusion is not to result. Thus, I use e_v for grid-to-reference-node (ground) voltage and e_a for plate-to-ground voltage. I would prefer to use the more familiar symbols e_g and e_p ; however, these standard symbols refer voltage to the cathode. It is only in the special case that the cathode also happens to be the reference node. Thus the symbols e_g and e_p are, unfortunately, relatively useless. In many diagrams, I use rather arbitrary symbols for e_v and e_a , such as e_0 , e_3 , and e_L , which avoids a cumbersome double-subscript notation. All too often, a strict adherence to standards is equivalent to a strait jacket.

In addition, I have not used the customary symbols for voltage and current generators. It is indeed unfortunate if the student cannot tell the two apart without an extra label. The convention showing a circle with an arrow alongside is particularly annoying (same symbol for both current and voltage); it breaks down completely when applied to, say, a source of force or velocity. The symbols I have adopted are unique, include polarity, do not require an extra label, clutter up a diagram to the least possible extent, and have an obvious interpretation; a current

source is a circle with an arrow inside, and a voltage source is a circle with plus and minus signs inside.

Little in the way of specific numerical designs will be found here except in the problems. The intent has been to present general methods from which any specific design is merely a straightforward application of the more general theory. This philosophy has been extended to the figures as well; where specific tube-characteristic and other curves might have been employed, generalized and qualitative sketches have been given instead. In any event, the inclusion of numerical examples should be the responsibility of the teacher and problem assignments rather than the text.

A separate bibliography has been provided with comments on the references in a chapter-by-chapter fashion. This obviates the need for footnotes and permits a more detailed account to be given of the scope of the various references as related to the material here.

Analyses with poles and zeros interpreted as phasors are little documented. Much of the work here is a result of personal effort and as such may show ignorance of what may have been done by others at an earlier date. The origin of the pole-zero method is difficult to ascertain; it may have been employed as a conceptual tool by such men as Gauss and Maxwell. Records of relatively early applications to circuit theory do not seem to be available except in association with normal modes and in analogy to potential functions. Neither of these interpretations has been emphasized here.

The origin of this book stemmed from conversations between Dr. D. O. Pederson, University of California, and myself in early 1953. From its very inception, Dr. H. H. Skilling, Chairman of the electrical engineering department at Stanford University, has provided considerable encouragement and enthusiasm, without which this book probably would not have been written. I am grateful to Professors J. M. Pettit and D. F. Tuttle, Jr., who introduced me to pole-zero concepts while I was a student at Stanford University in 1948-1949. Faculty members at the University of Michigan have been helpful in their comments on and enthusiasm for the work; in particular, I am grateful to Professor S. S. Attwood, Chairman of the electrical engineering department, who made typing facilities available for earlier versions of the text. Most of all, I am indebted to my wife Rita for her encouragement and patience through many long evenings of work.

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